The First IUGC Conference

Organizers: Yuichiro Hoshi (RIMS, Kyoto University)
Fumiharu Kato (IUGC, Tokyo Institute of Technology (PE))
Shinichi Mochizuki (RIMS, Kyoto University)

Dates: April 2 – April 5, 2024

Venue: Floor12, Dwango Seminar Room KABUKIZA TOWER, 4-12-15 Ginza, Chuo-ku, Tokyo, 104-0061 Japan

Schedule: Opening ceremony from 12:30, April 2

Lunch from 11:30 to 13:00 (12:30 on April 2) at the seminar room

	April 2 (Tues.)	April 3 (Wed.)	April 4 (Thu.)	April 5 (Fri.)
10:00 – 11:30		Short communications	Free discussion	Free discussion
11:30 – 13:00	Lunch (Opening from 12:30)	Lunch	Lunch	Lunch
13:00 – 14:00	Frobenioids (Porowski)	[OvEssLgcI] (Minamide)	[OvEssLgcV] (Yamashita)	[CbGT] (Minamide)
14:20 – 15:20	[Gateway] (Mochizuki)	[OvEssLgcII] (Minamide)	[OvEssLgcVI] (Yamashita)	[CbGal] (Tsujimura)
15:40 – 16:40	IUT (Lepage)	[OvEssLgcIII] (Minamide)	[OvGalOrbI] (Hoshi)	[RNSPM] (Tsujimura)
17:00 – 18:00	[ExpEst] (Porowski)	[OvEssLgcIV] (Yamashita)	[OvGalOrbII] (Hoshi)	Q & A (Mochizuki)

Program

April 2 (Tuesday)

13:00 – 14:00 Wojciech Porowski (RIMS, Kyoto University)
Model Frobenioids

14:20 – 15:20 Shinichi Mochizuki (RIMS, Kyoto University)
Inter-universal Teichmüller Theory as an Anabelian Gateway to Diophantine Geometry and Analytic Number Theory (IUGC2024 Version)

15:40 – 16:40 Emmanuel Lepage (Sorbonne University)
The logical structure of IUT (tentative)

17:00 – 18:00 Wojciech Porowski (RIMS, Kyoto University)
Explicit estimates in IUT

April 3 (Wednesday)

10:00 – 10:50 Taylor Dupuy (University of Vermont)
Interpretations in Anabelian Geometry

11:00 – 11:30 James Douglas Boyd (University of Western Ontario)
Philosophical Perspectives on Inter-Universal Teichmüller Theory

13:00 – 14:00 Arata Minamide (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory I

14:20 – 15:20 Arata Minamide (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory II

15:40 – 16:40 Arata Minamide (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory III

17:00 – 18:00 Go Yamashita (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory IV

April 4 (Thursday)

13:00 – 14:00 Go Yamashita (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory V

14:20 – 15:20 Go Yamashita (RIMS, Kyoto University)
On the essential logical structure of inter-universal Teichmüller theory VI

15:40 – 16:40 Yuichiro Hoshi (RIMS, Kyoto University)
On the Galois Orbit Version of Inter-universal Teichmüller Theory I

17:00 – 18:00 Yuichiro Hoshi (RIMS, Kyoto University)
On the Galois Orbit Version of Inter-universal Teichmüller Theory II

April 5 (Friday)

13:00 – 14:00 Arata Minamide (RIMS, Kyoto University)
The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space

14:20 – 15:20 Shota Tsujimura (RIMS, Kyoto University)
Combinatorial construction of the absolute Galois group of the field of rational numbers

15:40 – 16:40 Shota Tsujimura (RIMS, Kyoto University)
Resolution of nonsingularities and its application to the absolute Grothendieck conjecture

17:00 – 18:00 Shinichi Mochizuki (RIMS, Kyoto University)
Q & A

Abstracts

Speaker: Yuichiro Hoshi

Title: On the Galois Orbit Version of Inter-universal Teichmüller Theory I, II

Abstract: In these two talks, I explain some technical aspects of work in progress concerning a new enhanced versions of inter-universal Teichmüller theory, i.e., the Galois orbit version of inter-universal Teichmüller theory.

Speaker: Emmanuel Lepage

Title: The logical structure of IUT (tentative)

Abstract: TBA

Speaker: Arata Minamide

Title: On the essential logical structure of inter-universal Teichmüller theory I, II, III

Abstract: In this series of three talks, we will give a brief review of various important notions appearing in inter-universal Teichmüller theory and discuss the essential logical structure of the theory.

Speaker: Arata Minamide

Title: The Grothendieck-Teichmüller group as an open subgroup of the outer automorphism group of the étale fundamental group of a configuration space

Abstract: Let $n \geq 2$ be an integer and k an algebraically closed field of characteristic zero. Write Π_n for the étale fundamental group of the n-th configuration space of the projective line minus $\{0, 1, \infty\}$ over k, GT for the (profinite) Grothendieck-Teichmüller group, and $\mathfrak{S}_{n+3}$ for the symmetric group on $n+3$ letters. In this talk, we will discuss a result to the effect that the natural outer actions of GT and $\mathfrak{S}_{n+3}$ on Π_n determine an isomorphism $\mathrm{GT} \times \mathfrak{S}_{n+3} \xrightarrow{\sim} \mathrm{Out}(\Pi_n)$. This leads to a simple purely group-theoretic characterization of GT. This is joint work with Yuichiro Hoshi and Shinichi Mochizuki.

Speaker: Shinichi Mochizuki

Title: Inter-universal Teichmüller Theory as an Anabelian Gateway to Diophantine Geometry and Analytic Number Theory (IUGC2024 Version)

Abstract: One question that is frequently asked concerning *inter-universal Teichmüller theory* (IUT) is the following:

Why/how does IUT allow one to apply *anabelian geometry* to prove *diophantine* results?

In this talk, we address this question from various points of view. First, we discuss the fundamental framework underlying the relationship established by IUT between anabelian geometry, on the one hand, and diophantine geometry/analytic number theory, on the other. This discussion centers around the N-th power map on a subring of a field and the difference between regarding a group as a *Galois group*, on the one hand, and as an *abstract group* that is not equipped with an embedding into the automorphism group of a field, on the other. Here, we emphasize that this discussion is *entirely elementary* and only assumes a knowledge of *groups/monoids, rings, fields*, and the elementary geometry surrounding the *projective line/Riemann sphere*. We also briefly discuss certain

(again entirely elementary!) *set-theoretic/foundational subtleties* surrounding the notion of a "*gluing*". Such subtleties include the importance of working with "types/packages of data" called "*species*" (as opposed to underlying sets!), as well as the importance of obtaining "*closed loops*" of such types/packages of data in order to establish set-theoretic conclusions. Classical instances of such subtleties include the *conjugacy indeterminacies* inherent in the construction of the *algebraic closure* of a field and the closely related use of *norms* in Galois theory, as well as the classical notions of *analytic continuation/Riemann surfaces* (which is reminiscent of the classical dispute between *Riemann* and *Weierstrass*!) and *geodesic completeness/closed geodesics*. We then proceed to survey recent developments (work in progress) in IUT, many of which are closely related to the *Section Conjecture* in anabelian geometry for arbitrary hyperbolic curves over number fields. We also briefly mention recent progress on the Section Conjecture for hyperbolic curves over p-adic local fields, which is of interest in that it is closely related to the use of *Raynaud-Tamagawa "new-ordinariness"* in recent results on "*RNS*" (i.e., "resolution of nonsingularities"), in a fashion that may be regarded as a sort of p-adic local analogue of IUT. In the case of the Section Conjecture for hyperbolic curves over number fields, recent progress is closely related to 3 new enhanced versions of IUT that are currently under development. One of these new enhanced versions, namely, the *Galois-orbit version* of IUT, has new applications not only to the *Section Conjecture* for hyperbolic curves over number fields, but also to the *nonexistence of Siegel zeroes of certain Dirichlet L-functions*. The application to the Section Conjecture is interesting in that it exhibits and reconfirms the *essentially anabelian content of IUT*, i.e., as a *theory based on anabelian geometry that is applied to prove new results in anabelian geometry*. On the other hand, these recent applications, taken together with the original application of IUT to the ABC/Szpiro/Vojta Conjectures, are also noteworthy in that they may be regarded as a striking example of Poincaré's famous quote to the effect that

"*mathematics is the art of giving the same name to different things*".

That is to say, the *common name "IUT"* that may be regarded as describing, in essence, a *single mathematical phenomenon* that manifests itself, depending on relatively inessential (!) differences of context, as various (at first glance, unrelated!) *diverse phenomena* in *anabelian geometry*, *diophantine geometry*, and *analytic number theory*. The relationship with Poincaré's famous quote is also fascinating in that it was apparently motivated by various mathematical observations on the part of Poincaré concerning the similarities between *transformation group symmetries of modular functions such as theta functions* and *symmetry groups of the hyperbolic geometry of the upper half-plane* — all of which are topics (cf. the discussion above of Galois groups versus abstract groups!) that bear a profound relationship to IUT.

Speaker: Wojciech Porowski

Title: Explicit estimates in IUT

Abstract: In this talk we will discuss the content of the article 'Explicit estimates in Inter-universal Teichmuller theory'. In the original version of IUT some Diophantine inequalities were obtained; however constants involved were not explicit. We will show how a small modification allows one to apply the local part of IUT to all finite places, including primes of even residue characteristic. As a result, we can obtain an explicit version of these inequalities in the case of imaginary quadratic and rational number fields.

Speaker: Wojciech Porowski

Title: Model Frobenioids

Abstract: In this talk we will introduce the notion of a model Frobenioid. It is an abstract category which may be thought of as the category of line bundles on covers of some fixed base. We will describe examples of Frobenioids which appear in IUT and discuss how Kummer theory is used to relate 'Frobenius-like' structures with their 'etale-like' counterparts.

Speaker: Shota Tsujimura

Title: Combinatorial construction of the absolute Galois group of the field of rational numbers

Abstract: The absolute Galois group of the field of rational numbers ($G_{\mathbb{Q}}$) may be regarded as one of the central objects in modern number theory. One traditional approach [suggested by A. Grothendieck] to understanding $G_{\mathbb{Q}}$ is by studying the geometric Galois action of $G_{\mathbb{Q}}$ on the étale fundamental groups of suitable geometric objects. In this context, in light of Belyi's theorem, it is well-known that $G_{\mathbb{Q}}$ may be embedded into the Grothendieck-Teichmüller group GT [which was originally introduced by V. Drinfeld] via the natural outer action on the étale fundamental group of the projective line minus three points over the field of algebraic numbers. In light of this embedding, GT has been regarded as a sort of purely combinatorial approximation of $G_{\mathbb{Q}}$. On the other hand, it is totally unknown how precise this approximation is. In this talk, I will discuss recent progress surrounding this topic that is based on an approach via combinatorial anabelian geometry. In particular, we discuss a combinatorial construction of the conjugacy class of the purely arithmetic object $G_{\mathbb{Q}}$ in the purely combinatorial object GT, thus giving an explicit answer to a central question posed by G. V. Belyi. This is joint work with Y. Hoshi and S. Mochizuki.

Speaker: Shota Tsujimura

Title: Resolution of nonsingularities and its application to the absolute Grothendieck conjecture

Abstract: Resolution of nonsingularities (RNS) may be regarded as a highly sophisticated version of the technique of "passing to a covering with singular reduction of a given curve with smooth reduction over a p-adic local field" that appears in classical work of S. Mochizuki in the 1990s. Subsequent to S. Mochizuki's work, A. Tamagawa developed a theory of RNS for the stable models of arbitrary hyperbolic curves over certain complete discrete valuation fields [including p-adic local fields] in 2004. On the other hand, E. Lepage developed a theory of RNS for arbitrary semi-stable models of hyperbolic Mumford curves over p-adic local fields in 2013. Recently, in a joint work with S. Mochizuki, we generalized these results and developed a theory of RNS for arbitrary semi-stable models of arbitrary hyperbolic curves over p-adic local fields. In this talk, we first discuss the definition of RNS and outline our construction. We then discuss an application of RNS to an absolute version of the Grothendieck conjecture for arbitrary hyperbolic curves over p-adic local fields (Lepage-Mochizuki-T.), which settles one of the major open questions in anabelian geometry.

Speaker: Go Yamashita

Title: On the essential logical structure of inter-universal Teichmüller theory IV, V, VI

Abstract: TBA

Short communications

Speaker: Taylor Dupuy (University of Vermont)

Title: Interpretations in Anabelian Geometry

Abstract: This talk is an introduction to the model theoretic formalism of interpretations and how they can (or sometimes can't) be used to discuss Mochizuki's functorial algorithms in IUT. Many examples will be given.

Speaker: James Douglas Boyd (University of Western Ontario)

Title: Philosophical Perspectives on Inter-Universal Teichmüller Theory

Abstract: Since the inception of Inter-Universal Teichmüller Theory (IUTT), much activity among the international mathematics community has been dedicated to gaining an understanding of IUTT and scrutinizing its proof of the Szpiro/abc/Vojta conjectures. Considerably less effort has been dedicated to developing a discourse on the theoretical contributions of IUTT itself and the new directions in which it takes Diophantine geometry. Thanks to the distribution of further expository work on IUTT in recent years, doing so is increasingly feasible. In what follows, we will consider implications of IUTT for the philosophy of mathematics. We do so with two aims in mind. The first is to articulate key themes in IUTT as made intelligible against longstanding discursive threads in philosophy. Such themes should be accessible to a broader readership. The second is to effectuate advances in the philosophy of mathematics itself, which is often underdeveloped with respect to contemporary topics in number theory and Diophantine/arithmetic geometry. Given the absence of philosophical discourse on topics such as anabelian geometry, Diophantine geometry, and scheme theory, it will be necessary to present philosophical framings of mathematical precedents to IUTT in order to address IUTT itself. What is more, due to a poverty of available philosophical literature, we will draw upon concepts and discourses from theoretical computer science, the philosophy of physics, and analytic philosophy in order to anchor our discussions in discursive precedent.